NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute Affiliated to AKTU, Lucknow)
M.Tech (Integrated)
SEM: III - THEORY EXAMINATION (2023 -2024)
Subject: Engineering Mathematics-III

Time: 3 Hours   **Max. Marks: 100**

General Instructions:
IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*
*1. This Question paper comprises of **three Sections -A, B, & C.** It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. Maximum marks for each question are indicated on right -hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A   20

1. Attempt all parts:-

1-a. $\lim_{z \to 0} \frac{z}{\bar{z}}$ (CO1)   1

    (a)   Limit exists
    (b)   Limit does not exist
    (c)   Limit exists and equal to 1
    (d)   None of these

1-b.   Analytic function is also known as (CO1)   1

    (a)   Wavelet function
    (b)   Holomorphic function
    (c)   Hazard function
    (d)   None of these

1-c.   If $f(z) = \frac{\sin z}{z}$, then $z = 0$ is   1

    (CO2)

    (a)   Removable singularity
    (b)   Non-Isolated singularity
    (c)   Essential singularity
    (d)   None of these

1-d. The singular points of $f(z) = \dfrac{1}{z(z-1)^2}$ are (CO2)   1

(a) 0,1,-1

(b) 0,1,1

(c) 1,-1

(d) None of these

1-e. A partial differential equation requires :(CO3)   1

(a) Exactly one independent variable

(b) Two or more independent variables

(c) Can't Say

(d) None of these

1-f. Solution of the PDE $D'(D + 2D' + 1)Z = 0$ is (CO3)   1

(a) $z = f_1(y) + f_2(x)$

(b) $z = f_1(y) + f_2(y + 2x)$

(c) $z = f_1(x) + e^{-x} f_2(y - 2x)$

(d) None of these

1-g. Which method is used for finding roots of equations (CO4)   1

(a) Bisection Method

(b) Newton Raphson Method

(c) Regula Falsi Method

(d) All of the above

1-h. Interpolation formula for unequally spaced values of argument is: 1

(CO4)

(a) Newton's Forward Interpolation

(b) Newton's Backward Interpolation

(c) Newton's Divided Difference Interpolation

(d) None of these

1-i. A can complete a certain work in 4 minutes, B in 5 minutes, C in 6 minutes, D in 10 minutes and E in 12 minutes. The average number of units of work completed by them per minute will be   1

(CO5)

(a) 0.16

(b) 0.40

(c) 0.80

(d) None of these

1-j. A train is moving with a speed of 90 km/hr. How many metres will it cover in 10 minutes? (CO5)   1

(a)   16 Km

(b)   17 Km

(c)   15 Km

(d)   1540 Km

2. Attempt all parts:-

2.a.   Check that the $\displaystyle \lim_{z \to 0} \frac{z}{\bar{z}}$ is exists or not ?   (CO1)   2

2.b.   Evaluate: $\displaystyle \oint_c \frac{z^2 - z + 1}{z - 2} dz$; C: $|z - 1| = \frac{1}{2}$.   (CO2)   2

2.c.   Solve the PDE: $(D - 5D' + 1)^3 z = 0$.   2

(CO3)

2.d.   Write Simpson's Rule. (CO4)   2

2.e.   12 buckets of water fill a tank when the capacity of each bucket is 13.5 litres. How many buckets will be needed to fill the same tank, if the capacity of each bucket is 9 litres?   (CO5)   2

SECTION-B   30

3. Answer any <u>five</u> of the following:-

3-a.   Determine an analytic function $f(z)$ in terms of z whose real part is $e^{-x}(x \sin y - y \cos y)$.   6

(CO1)

3-b.   Find the bilinear transform which maps the points $z = 0, -1, i$ into the points $w = i, 0, \infty$ respectively.(CO1)   6

3-c.   Evaluate $\displaystyle \oint_C \frac{1}{z^2(z^2 + 9)} dz$; where C is a circle $|z| = 2$.   6

(CO2)

3-d.   Determine the pole and residue at each pole of the function $f(z) = \dfrac{z^2}{(z - 2)(z - 1)^2}$.   6
     (CO2)

3.e.   Find the solution of the one-dimension wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$, which is consistent with the physical nature of the problem. (CO3)   6

3.f.   Find a positive value of $(17)^{1/3}$ correct to the five decimal places by Newton Raphson Method. (CO4)   6

3.g.   Two pipes X and Y fill a tank in 15 hrs. and 20 hrs. respectively, while a third pipe 'Z' can empty the full tank in 25 hrs. All the three pipes are opened in the beginning. After 10 hrs. Z is closed. In how much time, will the tank be full? (CO5)   6

SECTION-C   50

4. Answer any <u>one</u> of the following:-

4-a. $f(z) = \dfrac{x^3 y^5 (x + iy)}{x^6 + y^{10}}, \quad z \neq 0 \; ; \; f(0) = 0$ 10

Show that the function defined by   is not
analytic at the origin even though it satisfies Cauchy-Riemann equations at the
origin.
 (CO1)

4-b.   If f(z) is an analytic function then prove that (CO1)   10

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|\mathbf{Re}f(z)|^2 = 2|f'(z)|^2$$

5. Answer any <u>one</u> of the following:-

5-a.   Expand $\dfrac{1}{(z+1)(z+3)}$ in the region: 10

(i) $|z| < 1$

(ii) $1 < |z+1| < 2$ (CO2)

5-b.   Determine the poles of the following function and residues at each pole: 10

$f(z) = \dfrac{(z-1)}{(z+1)^2(z-2)}$ and hence evaluate $\oint_C f(z)dz$, where C is the circle

$|z - i| = 2$. (CO2)

6. Answer any <u>one</u> of the following:-

6-a.   Solve the PDE: $4r + 12s + 9t = e^{3x-2y}$.(CO3)   10

6-b.   Solve the PDE: $4\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = 3u$ subject to the condition $u(0, y) = 4e^{-y} - e^{-5y}$ 10

by method of separation of variables.
 (CO3)

7. Answer any <u>one</u> of the following:-

7-a.   Find the real root of x^3-2x-5=0 correct to three decimal places using Newton-Raphson method.   10

          (CO4)

7-b.   Apply Runge Kutta method to find an approximate value of y when $x = 0.2$, given that   10

(CO4)
$dy/dx = x + y$, $y = 1$ when $x=0$.

8. Answer any <u>one</u> of the following:-

8-a.   (i) Bucket P has thrice the capacity as bucket Q. It takes 60 turns for bucket P to fill the empty drum. How many turns will it take for both the buckets P and Q,   10

having each turn together to fill the empty drum?

(ii) Two pipes A and B can fill a tank in 15 hours and 20 hours respectively while a third pipe C can empty the full tank in 25 hours. All the three pipes are opened in the beginning. After 10 hours, C is closed. In how much time will the tank be full?   (CO5)

8-b.   (i) A can do a work in 4 days, B in 5 days and C in 10 days. Find the time taken by   10
A, B and C to do the work together?

(ii) If Roger can do a piece of work in 8 days and Antony can complete the same work in 5 days, in how many days will both of them together complete it? (CO5)